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Observing Gravitational Waves from a Black Hole Merger Event

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ABSTRACT

In late 2015, detectors at both the Washington and Louisiana installations of Laser Interferometer Gravitational-Wave Observatory (henceforth abbreviated as: LIGO) simultaneously observed a gravitational wave for the first time. These waves were produced from a merger of two black holes, of masses $36^{+5}_{-4}M_{\odot}$ and $29^{+4}_{-4}M_{\odot}$. These gravitational waves also happened to match the expected frequency of two black holes spiralling into one another and merging. The final mass of the black hole was calculated to be $62^{+4}_{-4}M_{\odot}$ which is less than the sum of the two original black holes, this is due to radiation of the gravitational wave in question by the relation $E = Mc^2$ and corresponds to a total mass radiated of $3.0^{+0.5}_{-0.5}M_{\odot}$ [3]

Key words. gravitational waves – black holes – LIGO – black hole merger – chirp mass

1. Introduction

Black holes are regions of space which have so much mass that the gravitational pull of the body will not allow even light to escape. To put it more precisely, they are regions of space that have a mass density such that you must exceed the speed of light to escape the gravitational attraction of it. Mathematically this can be defined as body which has an escape velocity equal to c, the speed of light, the point of the black hole in-which the escape velocity is exactly c is known as the event horizon or the surface of the black hole. These massively dense objects are formed from the collapse of massive stars ($M > 8M_{\odot}$) which result in type II supernovae, with the notable exception of primordial black holes (which have been in existence since the very early universe) and those type II supernovae that result in neutron stars.[4]

Much like how electromagnetic waves are created by and propagate from oscillating charges (time-dependent electric dipoles). Gravitational waves are created by and propagate from time-dependant mass-energy quadropoles (quadrupoles being comprised of four equal monopoles or two equal dipoles), for example: orbiting binary stars, neutron stars, black holes or any other. [4] Since these are waves in space-time they bend and stretch the universe itself. By measuring these very very small perturbations in space-time, gravitational wave observatories can measure the amplitude and frequencies of these waves and then can use these values to back calculate attributes of the source of the waves.

The first predictions of the existence of gravitational waves go back to Einstein's work in the early 20th century, shortly after he finished formulating his field equations. He also had the understanding that these waves would have extraordinarily small amplitudes, as expected given the nature of gravity being such a weak force. [3] However, he would later "un-predicted" it assuming that it was a by-product of his approximations made in

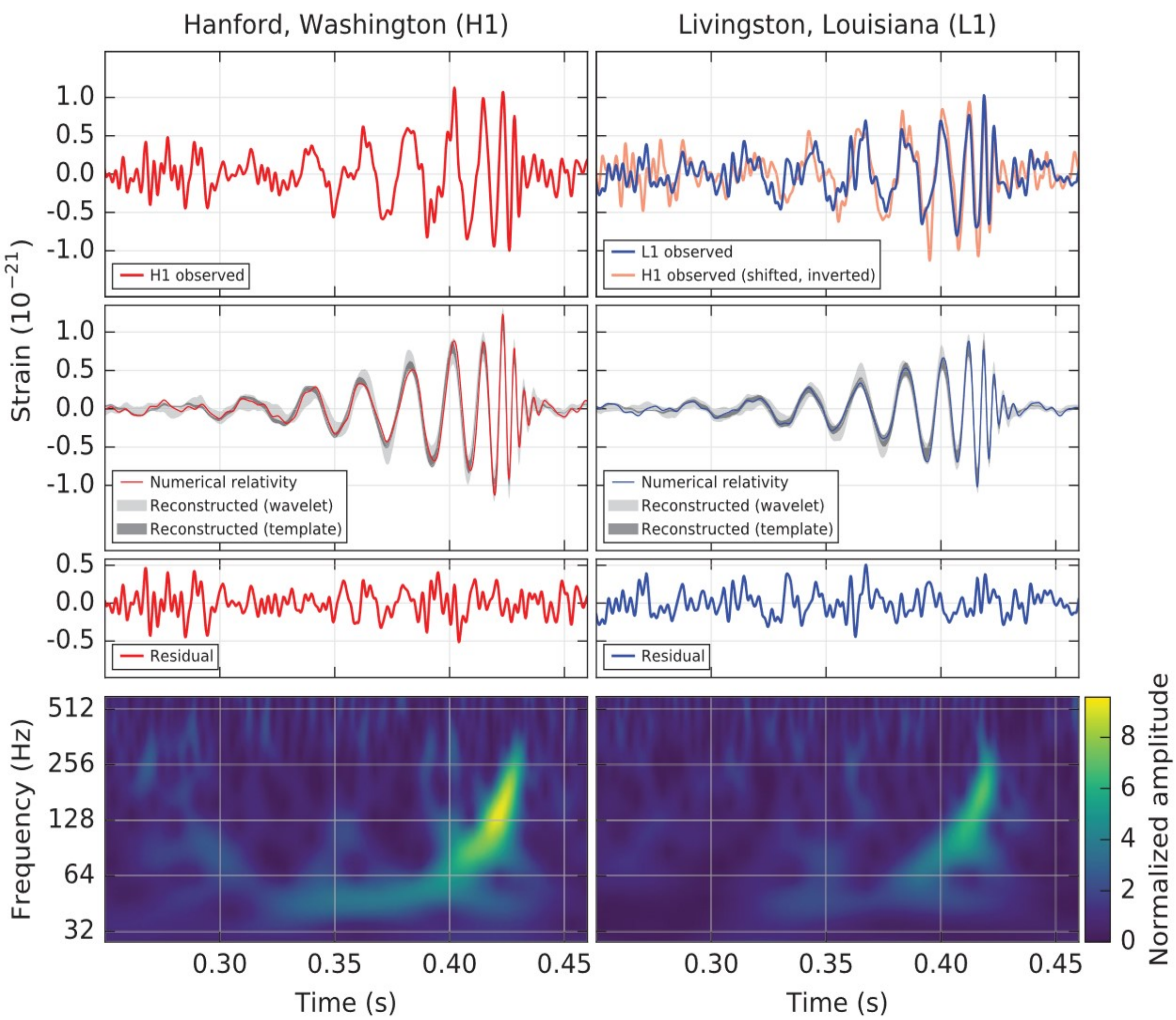

Fig. 1. *Top row*: the raw strain data from the Hanford, Washington site (hereafter abbreviated as H1) and the Livingston, Louisiana site (hereafter abbreviated as L1). The wave was observed first at L1 and then, approximately $6.9^{+0.5}_{-0.4}$ ms later, at H1. *Second row*: the same data as the above graph but with extra noise filtered out, isolating the gravitational wave pattern with 99.9% confidence based on the algorithm used in Abbott et al. [3] The gray areas are from other independent wave reconstructions to account for potential error with a 90% confidence. *Third row*: residuals after subtracting the expected model from the recorded data. [3] *Bottom row*: visual depiction of the frequency and amplitude of the wave with respect to time. [3]
Source: Abbott et al. [3]

earlier work. But, much like his cosmological constant Λ, despite his own doubts he was correct yet again.[4] Later, in 1982 Taylor and Weisberg studied a binary pulsar system and their findings of the energy loss of the system agreed strongly with Einstein's general relativity and it's prediction of gravitational waves. [5]

Yet still, gravitational waves had yet to be directly detected. This lead to the building of several facilities designed specifically to detect these waves in Japan (TAMA 300), Germany (GEO 600), Italy (Virgo) and the United States (LIGO). After over a decade of observations, fine tuning and sensitivity improvements of the detectors, finally in September of 2016, LIGO made the first direct detection of a gravitational wave. [3]

2. Data

Some important findings from Abbott et al. [3] can be found in Fig. 1, most notably that over the span of 0.2 s the signal goes from 35 Hz to 150 Hz over the course of 8 cycles, where the amplitude achieves its highest value, after-which the wave quickly dies off. [3] The fish diagrams at the bottom of Fig. 1 for the binary black hole merger as well as the fish diagrams for the

Table 1. Important findings from Abbott et al. [3]

Primary black hole mass	$36^{+5}_{-4} M_{\odot}$
Secondary black hole mass	$29^{+4}_{-4} M_{\odot}$
Final black hole mass	$62^{+4}_{-4} M_{\odot}$
Final black hole spin	$0.67^{+0.05}_{-0.07}$
Luminosity distance	410^{+160}_{-180}Mpc
Source redshift z	$0.09^{+0.03}_{-0.04}$

binary neutron star merger in Fig. 3 will also be important as we can formulate estimates of the frequency (f) and the change in frequency with respect to time ($\dot{f}$) from them.

3. Method

LIGO and other gravitational wave observatories utilize a specialized version of a Michelson interferometer specially designed to detect the incredibly weak signature of gravitational waves. Each arm of the LIGO interferometer is 4km long with a mirror at each end, a change to the standard Michelson interferometer is the two resonant optical cavities and the power-recycling mirrors in the interferometer. These additions serve the purpose of boosting any gravitational wave signals by increasing its effect on the light by a factor of 300, making any potential signals much easier to detect. [3]

With the arms each being a set distance we can use this baseline to measure and calculate the amplitude of any passing gravitational wave. Using the equality:

$$L = L_x = L_y = 4\text{km} \tag{1}$$

We also know that the gravitational wave, when propagating orthogonally passed the detector will cause one arm of the array to expand and one to contract at a distance proportional to the amplitude of the wave, satisfying the equality:

$$\Delta L = \delta L_x = \delta L_y = h(t)L \tag{2}$$

where L is the length of the arms and h is the amplitude of the gravitational wave stretching or compressing the arm.[3]

Another important aspect of LIGO is that both of its detectors are separated by thousands of kilometers, this way some local phenomena and local noise can be singled out for potential spikes. If both detectors pick up on the same exact signal at roughly the same time (as the wave is propagating at the speed of light it should be effectively identical times give or take a few milliseconds) then we can rule out a local phenomena as nothing else other than a gravitational wave could cause identical signatures to appear at both detectors simultaneously. This makes working with the other gravitational wave facilities important as well, if the same signature appears at both US detectors and then the Italian and Japanese detectors also pick up an identical signal, this is even stronger proof and confirmation that a gravitational wave was detected. [3]

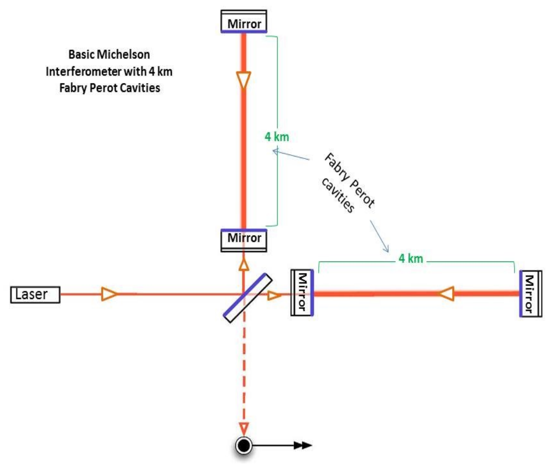

Fig. 2. Simplified diagram of the LIGO interferometer.
Source: Caltech LIGO

Once we obtain this data we can take the strain data and see how much the amplitude changes with time, the faster the amplitude changes with respect to time the higher the frequency of the oscillation. Once we obtain the frequency we can also plot this with respect to time (see: bottom row of Fig. 1) and measure how much the frequency changes with time. Once we have obtained the frequency and the change in frequency with respect to time we can calculate the chirp mass of the oscillations.

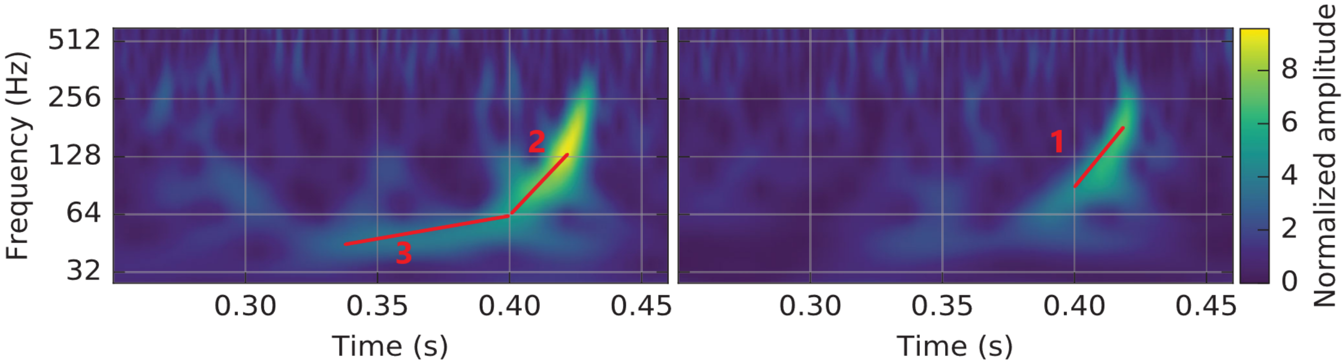

Fig. 3. Using the frequency and the change of the frequency with respect to time of lines 1, 2 and 3 here we can use in eq. 3 to estimate the chirp mass of the binary black holes. The main advantage to calculating the chirp mass is that it is much easier than trying to find the mass of black holes individually from the gravitational wave data.
Source: Abbott et al. [3]

4. Results

The chirp mass $\mathcal{M} \geq 30M_{\odot}$ which can be confirmed by the equation:

$$\mathcal{M} = \frac{(m_1 m_2)^{3/5}}{(m_1 + m_2)^{1/5}} = \frac{c^3}{G}\left[\frac{5}{96}\pi^{-8/3} f^{-11/3} \dot{f}\right]^{3/5} \tag{3}$$

Using the fish graph for L1 at the bottom of Fig. 1 (represented as line 1 in Fig. 3) we can estimate f and $\dot{f}$. For these estimates we will take f as being 125 Hz, where the graph amplitude peaks, and estimating $\dot{f}$ by taking $\frac{\Delta f}{\Delta t}$ between 0.4 s and 0.43 s with corresponding frequency estimates of 100 Hz and 150 Hz respectively. Using the equation above, these will give us a chirp mass value of $30M_{\odot}$ which is identical to the value for the chirp mass found in Abbott et al. [3]

To further confirm these results we can take and get an idea for the error we can take two more lines from the H1 fish graph (lines 2 and 3 in Fig. 3). Starting at the higher frequency we can estimate f being around 85 Hz and as before approximating $\dot{f}$ as $\frac{\Delta f}{\Delta t}$ between 0.4 s and 0.43 s. This gives us a value of $\mathcal{M} = 31M_{\odot}$, fairly close to our value found for L1. For the lower end of the frequency spectrum, the H1 data still has enough of a signal for us to estimate. Finally for line 3 in Fig. 3, this time taking f to be about 55 Hz and taking $\frac{\Delta f}{\Delta t}$ between 0.34 s and 0.4 s we get a twitch mass value of around $\mathcal{M} = 28M_{\odot}$. This value is the same as found using the estimations of line 3 in Fig. 3, the value found by Abbott et al. was $30M_{\odot}$. The values found here correlate with those of Abbott et al. within an error of just a few solar masses.

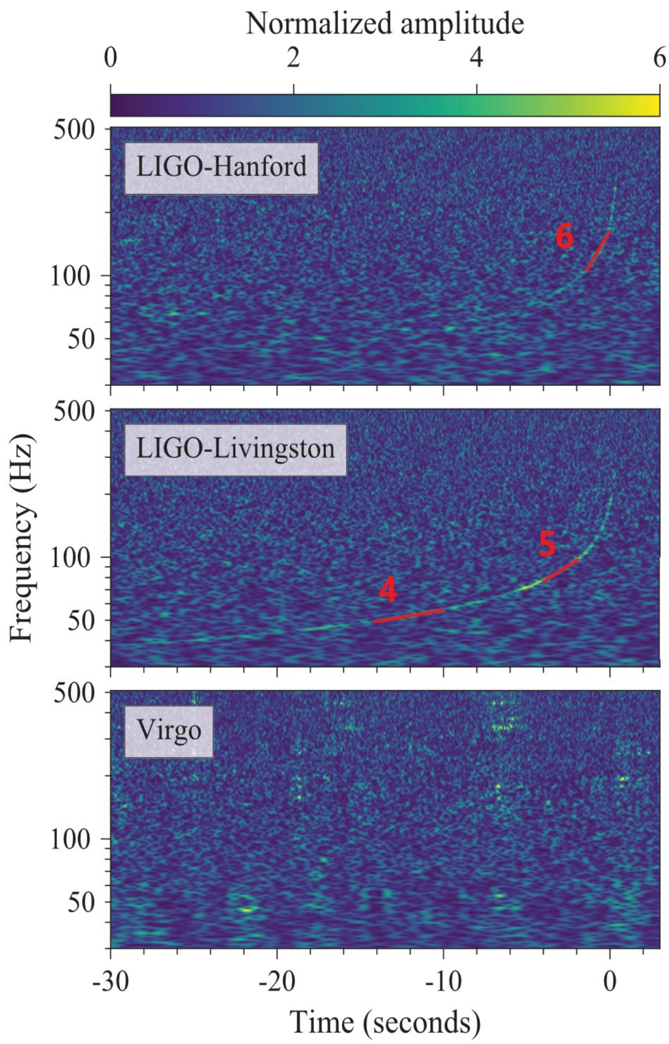

Fig. 4. Using line 4 we can estimate the chirp mass of this of the neutron star merger from the 2017 neutron star merger event.
Source: Abbott et al. [2]

We can also use the masses in Table 1 to substitute in for m_1 and m_2 in the chirp mass equation found above. If we do this we get a chirp mass of $\mathcal{M} = 28M_{\odot}$. To further prove that these were black holes and not some other astronomical bodies we will look at another set of gravitational waves from a neutron star merger in Abbott's 2017 paper. [2]

Using the L1 data from the neutron star merger event in 2017 (Fig. 4) we can calculate the chirp mass for this even as well. Once again estimating f and $\dot{f}$ as 50 Hz and by estimating the slope of line 4 in Fig. 4 we get a chirp mass of around $\mathcal{M} = 1.16M_{\odot}$. Using the same method for line 5 (Fig. 4) the chirp mass calculates to be $1.11M_{\odot}$ and for line 6 (Fig. 4) from the H1 data gives us a chirp mass of around $1.14M_{\odot}$ with estimated frequencies of 90 Hz and 125 Hz respectively. These are all fairly close (approx. 7% error) to the calculated chirp mass found in the paper of $\mathcal{M} = 1.188M_{\odot}$ [2], however this still serves to show that the chirp mass of the neutron star merger is an order of magnitude smaller than the one calculated from the 2016 gravitational wave data.

5. Discussion

The main thing we can take away from the findings in Fig. 1 is that the black holes were orbiting one another continually picking up speed and inspiralling closer and closer over time. Eventually the two black holes started inspiralling fast enough that the mass-energy radiation (gravitational waves) was intense enough to be detected by LIGO. In the span of 0.2 the black holes came within 350 km of one another, made contact with one another and then merged with one another. This process is beautifully illustrated in Fig. 3 from Abbott et al. [3]

A good way of picturing this merging and the following ringdown is to picture the black holes as two drops of water. Much like when the two drops meet each other and almost instantly merge together to form a larger drop. Though with the black holes this process is driven by the intense gravity of the objects whereas the water droplet merging is driven by the surface tension of the water that comprises the droplets. Nonetheless the image of the two is very similar, even if the physics are a bit different.

The gravitational waves indicate to us two important things: the speed of rotation of the two bodies was incredibly high and the two masses must have been immensely dense. In order to reach the orbital frequency that these two objects were oscillating, they must have been around 350 km apart. A system comprised of two neutron stars would not massive enough to reach the necessary mass (note that the upper limit of neutron stars is a bit more than $2.1M_{\odot}$) required to reach the calculated chirp mass as well as a neutron-black hole merger would merge at much lower speed and frequency as the black hole would completely dominate the oscillations of the binary system. [3]

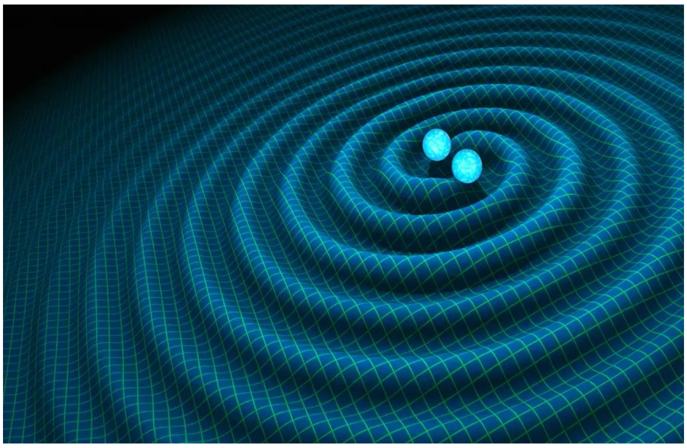

Fig. 5. Visual representation of gravitational waves emitted from the inspiral of two massive objects in a binary system.
Source: NASA

As expected by the nature of gravitational waves, being waves of mass-energy radiation, the resulting black hole after the merger is actually less than the total masses of the original two black holes. As noted in Table 1, the two black holes originally had masses of $36^{+5}_{-4}M_{\odot}$ and $29^{+4}_{-4}M_{\odot}$ which post merger without mass-energy radiation should be around $65M_{\odot}$ however the actual final black hole mass was found to be $62^{+4}_{-4}M_{\odot}$, which implies that the total energy of the gravitational wave is $3.0M_{\odot}c^2$ using the famous $E = Mc^2$, also confirming that the gravitational wave is a form of mass-energy radiation. Additionally, as the result of an inspiral merger of two black holes, the final black hole is a spinning black hole. Also known as a Kerr black hole. These black holes are defined by not only their mass like all black holes, but also their spin. This spin, also known as the Kerr metric, describes how much the black hole is spinning (a Kerr metric of 0 relates to no spin at all). [3]

When looking at the chirp mass calculated from the binary neutron star merger data the chirp mass is an order of magnitude smaller than the binary black hole merger. While these neutron stars, which are formed in the same way that black holes are formed, are also incredibly dense and massive, they do not have the necessary density needed to reach the frequencies reached by these waves while also having the corresponding mass needed for the chirp mass that was found. In essence, their density would require them to have a larger radius and thus could not orbit at the frequencies and distances observed if the mass is correct.

Once we look at the fish diagrams for the neutron star merger in the 2017 LIGO data, these differences become even more apparent. Where the black hole merger data had a large and wide area due to the uncertainty in the data, caused by how quickly the merging event occur, and a larger strain amplitude, the neutron star merger was a very thin line with less uncertainty and also reached a lower normalized amplitude at the peak of the merger. Also notice that the neutron stars took much longer to merge than the black holes, the change in frequency with respect to time is also much more intense for the black holes, though the neutron stars do reach an overall frequencies that are higher.

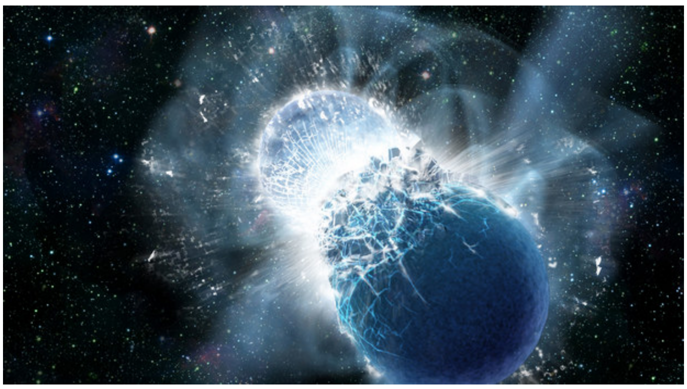

Fig. 6. Artist's depiction of a neutron star merger.
Source: LIGO Caltech

Fig. 7. A visualization of two black holes at the point of merging just, before ringdown into the final black hole.
Source: black-holes.org

6. Conclusions

The main conclusion is that LIGO did in fact detect gravitational waves and those waves were also the signature of a binary black hole merger. The data matches very well with the model as the residuals stay consistent throughout the entirety of the time window where the gravitational wave signature was observed. The H1 data matches more closely to the model than the L1 data, the L1 residuals have higher peaks when compared to the H1 data and has a larger area of error in the second row of Fig. 1. However, they both are still very close to the model which is why it is believed that this is indeed a gravitational wave signature and then when considering the frequency that the bodies reached before merging as well as the chirp mass, it must be a binary black hole merger.

Two major implications of these findings are the existence of black holes $\geq 25M_{\odot}$ as well as the fact the binary black holes can form and merge within a Hubble time. [3] Aside from gravitational waves, proving the existence of binary black holes has a profound impact on models that make predictions using the black hole merger rate, since we have now proven that binary black holes do in-fact exist, any model that does not allow for

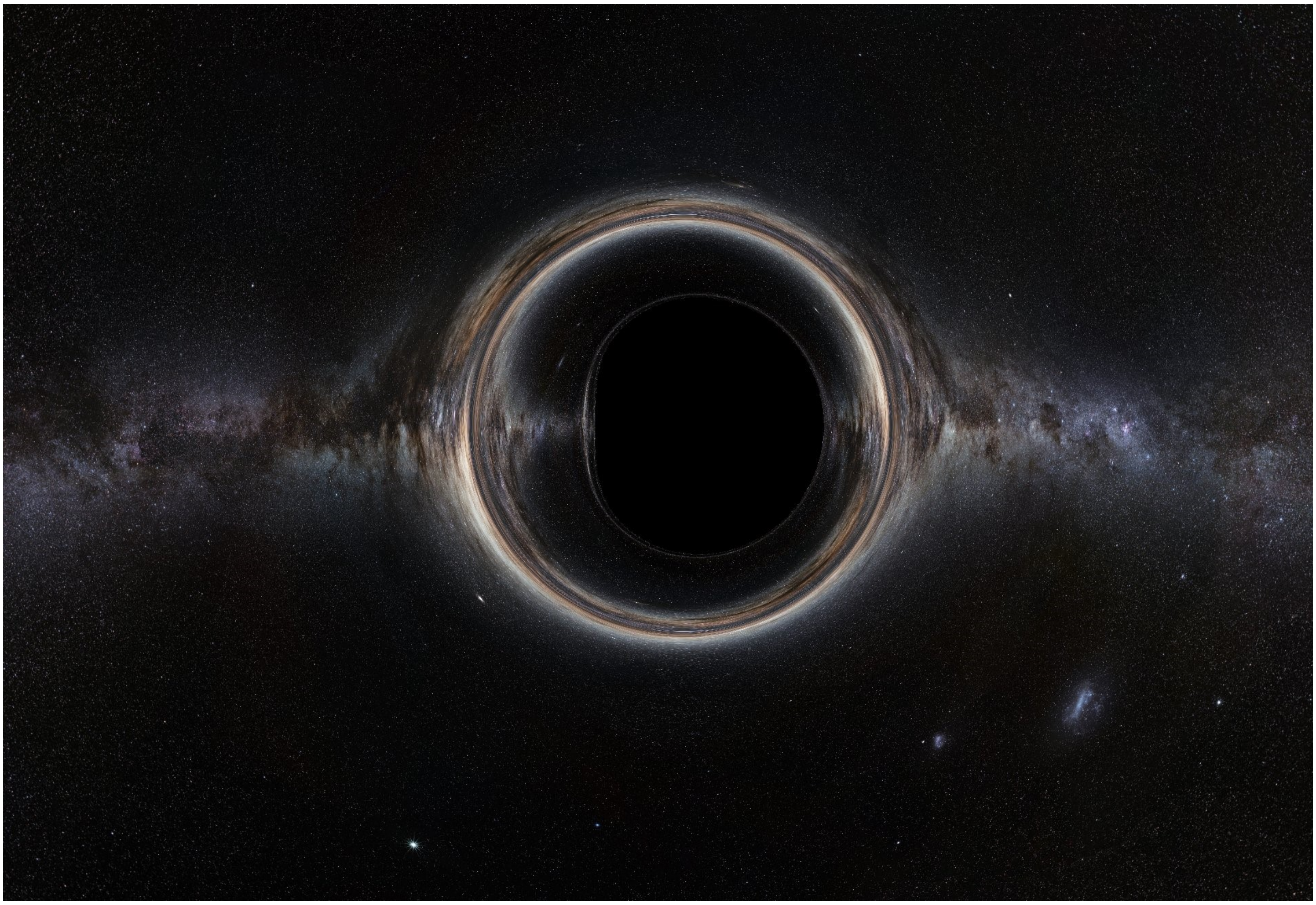

Fig. 8. A visualization of a Kerr black hole here placed in front of the galactic plane with with light from the core being gravitationally lensed around it. The black hole in this visualization is spinning from left to right with the left side spinning towards the viewer. This is what the final black hole would look like after the ringdown.
Source: ESA ACT black hole visualization

them to occur or does not allow for them to merge within a Hubble time can now be excluded. [1]

These findings are huge as it allows us to greatly reduce the number of models we have for the universe as well as for black hole formation and merging. This also must be a black hole merger as black holes are the only object that has enough mass to reach the chirp mass calculated, of similar enough mass to reach the frequency recorded in the data, while also being dense enough to have not collided and merged earlier in the inspiral thus allowing us to say that we can apply these findings towards black hole and black hole merger models. The same can be applied to the 2017 neutron star merger findings and models relating to neutron stars and neutron star mergers.

Now that these LIGO observations have proven the viability of detecting gravitational waves, future astronomical and astrophysical studies. The Laser Interferometer Space Antenna (LISA), the space based gravitational wave observatory, is now even more scientifically important. If LIGO and other earth based gravitational wave facilities had continued to not detect anything, it is likely LISA would have been in danger of being completely scrapped. With the added benefit of being in space and thus without the interference of perturbations from Earth based noise (such as the passing truck that caused some issues with the detection of the neutron star merger data for one of the observatories), LISA could potentially see gravitational waves from black hole mergers at higher redshifts possibly allowing us to see even larger black hole merger events that happened further back in time. This could even allow us to start to create a relationship for the rate of black hole mergers with respect to time, perhaps there will even be a relationship between large black hole mergers with the formation of galaxies and offer an insight into why galaxies have super massive black holes in their cores.

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